

MENIIT

NEET | IIT-JEE | FOUNDATION

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JEE MAIN-2022

COMPUTER BASED TEST (CBT)

DATE : 27-06-2022 (MORNING SHIFT) | TIME : (9.00 AM to 12.00 PM)

Duration 3 Hours | Max. Marks : 300

**QUESTIONS
&
SOLUTIONS**

PART : PHYSICS

1. A projectile is launched at an angle ' α ' with the horizontal with a velocity 20 ms^{-1} . After 10 s, its inclination with horizontal is ' β '. The value of $\tan\beta$ will be : ($g = 10 \text{ ms}^{-2}$).

- (A) $\tan\alpha + 5\sec\alpha$
 (B) $\tan\alpha - 5\sec\alpha$
 (C) $2\tan\alpha - 5\sec\alpha$
 (D) $2\tan\alpha + 5\sec\alpha$

Ans. (B)

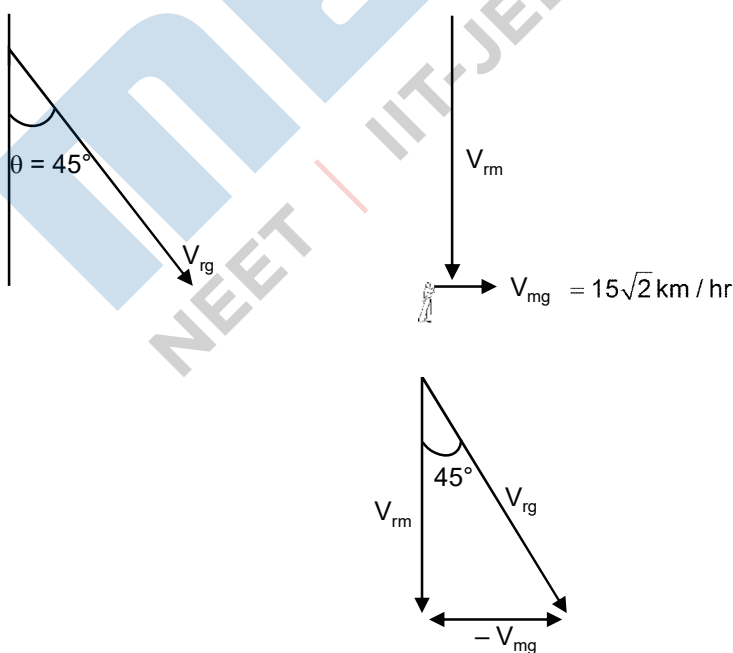
Sol. $\tan\beta = v_y/v_x = \frac{20\sin\alpha - 100}{20\cos\alpha} = \tan\alpha - 5\sec\alpha$

2. A girl standing on road holds her umbrella at 45° with the vertical to keep the rain away. If she starts running without umbrella with a speed of $15\sqrt{2} \text{ kmh}^{-1}$ the rain drops hit her head vertically. The speed of rain drops with respect to the moving girl is:

- (A) 30 kmh^{-1}
 (B) $\frac{25}{\sqrt{2}} \text{ kmh}^{-1}$
 (C) $\frac{30}{\sqrt{2}} \text{ kmh}^{-1}$
 (D) 25 kmh^{-1}

Ans. (C)

Sol.



$$V_{rm} = V_{rg} \cos 45^\circ$$

$$V_{mg} = V_{rg} \sin 45^\circ = 15\sqrt{2}$$

$$V_{rg} = 30 \text{ km/hr}$$

$$\therefore V_{rm} = \text{km/hr}$$

3. A silver wire has a mass (0.6 ± 0.006) g, radius (0.5 ± 0.005) mm and length (4 ± 0.04) cm. The maximum percentage error in the measurement of its density will be:
- (A) 4%
- (B) 3%
- (C) 6%
- (D) 7%

Ans. A

Sol. $M = \pi r^2 \ell \rho$

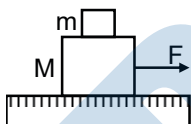
$$\rho = \frac{m}{\pi r^2 \ell}$$

$$\frac{\Delta \rho}{\rho} \times 100 = \left| \frac{\Delta m}{m} \right| \times 100 + 2 \left| \frac{\Delta r}{r} \right| \times 100 + \left| \frac{\Delta \ell}{\ell} \right| \times 100$$

$$\frac{0.006}{0.6} \times 100 + 2 \frac{0.005}{0.5} \times 100 + \frac{0.04}{4} \times 100$$

$$= 1 + 2 + 1 = 4\%$$

4. A system of two blocks of masses $m = 2$ kg and $M = 8$ kg is placed on a smooth table as shown in figure. The coefficient of static friction between two blocks is 0.5. The maximum horizontal force F that can be applied to the block of mass M so that the blocks move together will be:



- (A) 9.8 N
- (B) 39.2 N
- (C) 49 N
- (D) 78.4 N

Ans. (C)

Sol. Maximum acceleration of upper block $= \mu g = 0.5 \times 9.8 = 4.9$
 $F_{\max} = (2 + 8) \times 4.9 = 49\text{N}$

5. Two blocks masses 10 kg and 30 kg are placed on the same straight line with coordinates $(0, 0)$ cm and $(x, 0)$ cm respectively. The block of 10 kg is moved on the same line through a distance of 6 cm towards the other block. The distance through which the block of 30 kg must be moved to keep the position of centre of mass of the system unchanged is:
- (A) 4 cm towards the 10 kg block

- (B) 2 cm away from the 10 kg block
- (C) 2 cm towards the 10 kg block
- (D) 4 cm away from the 10 kg block

Ans. (B)

Sol.
$$\frac{m_1 \Delta x + m_2 \Delta x_2}{m_1 + m_2} = \Delta x_{cm}$$

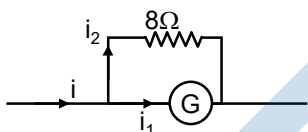
$$10(6) + 30(\Delta x_2) = 0$$

$$\Delta x_2 + 30(\Delta x_2) = 2 \text{ cm}$$

6. A 72Ω galvanometer is shunted by a resistance of 8Ω . The percentage of the total current which passes through the galvanometer is:
- (A) 0.1%
 - (B) 10%
 - (C) 25%
 - (D) 0.25%

Ans. (B)

Sol.



$$i_1 = \left(\frac{8}{8 + 72} \right) i = \frac{i}{10} \rightarrow \frac{i_1}{i} \times 100 = 10\%$$

7. Given below are two statements :

Statement I : The law of gravitation holds good for any pair of bodies in the universe.

Statement II : The weight of any person becomes zero when the person is at the centre earth.

In the light of the above statements, choose the correct answer from the options given below

- (A) Both **Statement I** and **Statement II** are true
- (B) Both **Statement I** and **Statement II** are false
- (C) **Statement I** is true but **Statement II** are false
- (D) **Statement I** is false but **Statement II** are true

Ans. (A)

Sol. At centre g is zero.

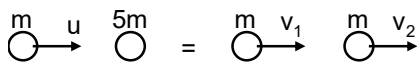
8. What percentage of kinetic energy of moving particle is transferred to a stationary particle when it strikes the stationary particle of 5 times its mass?

(Assume the collision to be head-on elastic collision)

- (A) 50.0%
- (B) 66.6%
- (C) 55.6%
- (D) 33.3%

Ans. (C)

Sol.



$$\begin{aligned}
 mu + 0 &= mv_1 + 5mv_2 \\
 v_1 + 5v_2 &= u \quad \dots\dots (1) \\
 v_2 - v_1 &= 1(u - 0) \quad \dots\dots (2) \\
 \text{add (1) \& (2)} \\
 6v_2 &= 2u \\
 v_2 &= u/3
 \end{aligned}$$

$$\frac{\frac{1}{2}(5m)v_2^2}{\frac{1}{2}mv^2} \times 100 = \frac{500}{9} = 55.6$$

9. The velocity of a small ball of mass 'm' and density d_1 , when dropped in a container filled with a glycerine, become constant after some time. If the density of glycerine is d_2 , then the viscous force acting on the ball, will be:

- (A) $mg\left(1 - \frac{d_1}{d_2}\right)$
- (B) $mg\left(1 - \frac{d_2}{d_1}\right)$
- (C) $mg\left(\frac{d_1}{d_2} - 1\right)$
- (D) $mg\left(\frac{d_2}{d_1} - 1\right)$

Ans. (B)

Sol. At the time of terminal velocity ball is in equilibrium

$$\begin{aligned}
 \text{i.e. } F_g &= F_b + F_v \\
 \Rightarrow F_v &= F_g - F_b \\
 \Rightarrow F_v &= Mg - g \\
 \Rightarrow mg &\left(\frac{d_2}{d_1} - 1\right)
 \end{aligned}$$

10. The susceptibility of paramagnetic material is 99. The permeability of the material Wb / A-m , is:
[Permeability of free space $\mu_0 = 4\pi \times 10^{-7} \text{ Wb / A-m}$]

- (A) $4\pi \times 10^{-7}$
(B) $4\pi \times 10^{-4}$
(C) $4\pi \times 10^{-5}$
(D) $4\pi \times 10^{-6}$

Ans. (C)

Sol. $\mu_r = 1 + \chi$
 $= 1 + 99 = 100$
 $\mu = \mu_r \mu_0 = 100 \times 4\pi \times 10^{-7}$
 $\mu = 4\pi \times 10^{-5}$

11. The current flowing through an ac circuit is given by

$$I = 5 \sin(122\pi t) \text{ A}$$

How long will the current take to reach the peak value starting from zero?

- (A) $\frac{1}{60} \text{ s}$
(B) 60 s
(C) $\frac{1}{120} \text{ s}$
(D) $\frac{1}{240} \text{ s}$

Ans. (D)

$$T = T = \frac{2\pi}{\omega} = \frac{2\pi}{120\pi} = \frac{1}{60}$$

$$\frac{T}{4} = \frac{1}{240} \text{ sec}$$

12. Match List - I with List - II :

- | | |
|----------------------|--|
| (a) ultraviolet rays | (i) study crystal structure |
| (b) Microwaves | (ii) Greenhouse effect |
| (c) Infrared waves | (iii) Sterilizing surgical instruments |
| (d) X-rays | (iv) Radar system |

Choose the correct answer from the options given below:

- (A) (a)-(iii), (b)-(iv), (c)-(ii), (d)-(i)
(B) (a)-(iii), (b)-(i), (c)-(ii), (d)-(iv)

(C) (a)-(iv), (b)-(iii), (c)-(ii), (d)-(i)

(D) (a)-(iii), (b)-(iv), (c)-(i), (d)-(ii)

Ans. (A)

Sol. Theory based.

13. An α particle and a carbon 12 atoms has same kinetic energy K . The ratio of their de-Broglie wavelengths ($\lambda_\alpha : \lambda_{C12}$) is :

(A) $1 : \sqrt{3}$

(B) $\sqrt{3} : 1$

(C) $3 : 1$

(D) $2 : \sqrt{3}$

Ans. (B)

Sol. $\lambda = \frac{h}{\sqrt{2mK}}$

$$\frac{\lambda_1}{\lambda_2} = \sqrt{\frac{m_2}{m_1}}$$

$$\frac{\lambda_1}{\lambda_2} = \sqrt{\frac{12}{4}}$$

$$\lambda_1 : \lambda_2 = \sqrt{3} : 1$$

14. A force of 10 N acts on a charged particle placed between two plates of a charged capacitor. If one plate of capacitor is removed, then the force acting on that particle will be.

(A) 5 N

(B) 10 N

(C) 20 N

(D) Zero

Ans. (A)

Sol. Initially force is $F = qE = q\left(\frac{q}{A \epsilon_0}\right) = \frac{q^2}{A \epsilon_0}$

after removing one plate force is

$$F' = qE' = q\left(\frac{q}{2A \epsilon_0}\right) = \left(\frac{q^2}{2A \epsilon_0}\right)$$

$$\Rightarrow F' = \frac{F}{2}$$

$$\Rightarrow F' = \frac{10}{2} = 5N$$

15. The displacement of simple harmonic oscillator after 3 seconds starting from its mean position is equal to half of its amplitude. The time period of harmonic motion is :
- (A) 6 s
 (B) 8 s
 (C) 12 s
 (D) 36 s

Ans. (D)

Sol. $X = A \sin \omega t$

$$\frac{A}{2} = A \sin \omega t$$

$$\omega t = \frac{\pi}{6}$$

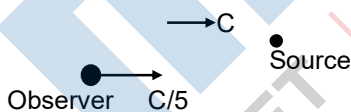
$$t = \frac{T}{12} = 3 \text{ sec.}$$

$$T = 36 \text{ sec.}$$

16. An observe moves towards a stationary source of sound with a velocity equal to one-fifth of the velocity of sound. The percentage change in the frequency will be:
- (A) 20%
 (B) 10%
 (C) 5%
 (D) 0%

Ans. (A)

Sol.



$$f' = \frac{V + V_0}{V} f$$

$$f' = \frac{C + C/5}{C} f$$

$$f' = \frac{6C}{5C} f$$

$$f' = \frac{6}{5} f$$

$$\frac{f' - f}{f} = \frac{6 - 5}{5}$$

$$\frac{\Delta f}{f} = \frac{1}{5}$$

$$\frac{\Delta f}{f} \times 100 = \frac{1}{5} \times 100 = 20\%$$

17. Consider a light ray travelling in air is incident into a medium of refractive index $\sqrt{2n}$. The incident angle is twice that a refracting angle. Then, the angle of incident will be :

(A) $\sin^{-1}(\sqrt{n})$

(B) $\cos^{-1}\left(\sqrt{\frac{n}{2}}\right)$

(C) $\sin^{-1}(\sqrt{2n})$

(D) $2\cos^{-1}\left(\sqrt{\frac{n}{2}}\right)$

Ans. (D)

Sol. $1\sin 2r = \sqrt{2n} \sin r$

$$2\sin r \cos r = \sqrt{2n} \sin r$$

$$\cos r = \sqrt{\frac{n}{2}}$$

$$r = \cos^{-1} \sqrt{\frac{n}{2}}$$

$$i = 2r = 2\cos^{-1} \sqrt{\frac{n}{2}}$$

18. A hydrogen atom in its ground state absorbs 10.2 eV of energy. The angular momentum of electron of the hydrogen atom will increase by the value of:

(Given, Planck's constant = 6.6×10^{-34} Js).

(A) 2.10×10^{-34} Js

(B) 1.05×10^{-34} Js

(C) 3.15×10^{-34} Js

(D) 4.2×10^{-34} Js

Ans. (B)

Sol. $\Delta E = 13.6 \times \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$

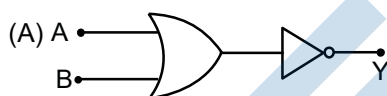
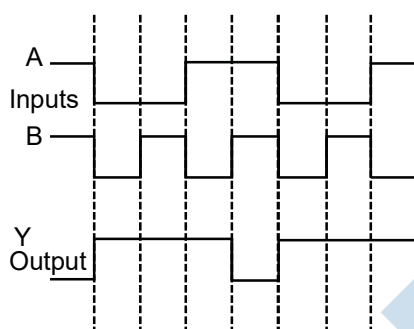
$$10.2 = 13.6 \times \left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right]$$

$$n_1 = 1 \text{ \& } n_2 = 2$$

change in angular momentum $\Delta_L = \frac{n_2 h}{2\pi} - \frac{n_1 h}{2\pi} = \frac{h}{2\pi}$

$$= 1.05 \times 10^{-34} \text{ J-s}$$

19. Identify the correct Logic Gate for the following out (Y) of two inputs A and B.



Ans. (B)

Sol. Truth table

A	B	Y
1	1	0
0	0	1
0	1	1
1	0	1
1	1	0
0	0	1

This truth table is possible in option (B).

20. A mixture of hydrogen and oxygen has volume 2000 cm^3 , temperature 300 K , pressure 100 kPa and mass 0.76 g . The ratio of number of moles of hydrogen to number of moles of oxygen in the mixture will be:

[Take gas constant $R = 8.3 \text{ JK}^{-1}\text{mol}^{-1}$]

(A) $\frac{1}{3}$

(B) $\frac{3}{1}$

(C) $\frac{1}{16}$

(D) $\frac{16}{1}$

Ans. (B)

Sol. $V = 2000 \text{ cm}^3$
 $P = 100 \text{ kPa}$
 $T = 300 \text{ K}$

Mass of gas mixture = $0.76 \text{ g (O}_2 + \text{H}_2)$

$$n = \frac{PV}{RT} = \frac{2 \times 10^{-3} \times 100 \times 10^3 \times 3}{25 \times 300}$$

$$n = \frac{2}{25}$$

$$n_2 = \frac{0.08(30 - 32 + 0.76)}{30}$$

$$\frac{n_1}{n_2} = \frac{0.08(32 - 0.76)}{0.08(30 - 32 + 0.76)} = \frac{32 - 0.76}{(-2 + 0.76)}$$

$$n_1(32) + n_2(2) = 0.76$$

$$n_2 + 16n_1 = 0.38$$

$$n_2 + n_1 = 0.08$$

$$15n_1 = 0.30$$

$$n_1 = \frac{0.30}{15} = 0.02$$

$$n_2 = 0.06$$

$$n_2/n_1 = 3$$

21. In a carnot engine, the temperature of reservoir is 527°C and that of sink is 200 K . If the workdone by the engine when it transfers heat from reservoir to sink is 12000 kJ , the quantity of heat absorbed by the engine from reservoir is _____ $\times 10^6 \text{ J}$.

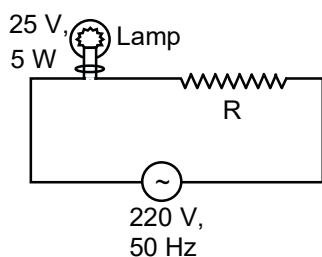
Ans. (16)

Sol. $\frac{W}{Q_1} = 1 = \frac{T_2}{T_1}$

$$\frac{12000}{Q_1} = 1 = \frac{200}{800}$$

$$Q_1 = 12000 \times \frac{4}{3} = 16000 \text{ kJ}$$

- 22.** A 220 V, 50 Hz AC source is connected to a 25 V, 5 W lamp and an additional resistance R in series (as shown in figure) to run the lamp at its peak brightness, then the value of R (in ohm) will be _____.



Ans. (975)

Sol. $i_{\text{rms}} = \frac{5}{25} = \frac{220}{R_B + R}$

$$R_B + R = 100$$

$$R_B = \frac{25 \times 25}{5} = 125$$

$$R = 1100 - 125 = 975$$

- 23.** In Young's double slit experiment the two slits are 0.6 mm distance apart. Interference pattern is observed on a screen at a distance 80 cm from the slits. The first dark fringe is observed on the screen directly opposite to one of the slits. The wavelength of light will be _____ nm.

Ans. (450)

Sol. According to question

$$\frac{\beta}{2} = 0.3\text{mm} \Rightarrow \beta = 0.6\text{mm}$$

$$\Rightarrow \frac{\lambda D}{d} = 0.6\text{mm} \Rightarrow \lambda = \frac{0.6d}{D} \Rightarrow \lambda = \frac{0.6 \times 0.6}{800}$$

$$\Rightarrow \lambda = 4.5 \times 10^{-4} \text{ mm} = 450 \text{ nm}$$

- 24.** A beam of monochromatic light is used to excite the electron in Li^{++} from orbit to the third orbit. The wavelength of monochromatic light is found to be $x \times 10^{-10}\text{m}$. The value of x is _____.

[Given $hc = 1242 \text{ eV nm}$]

Ans. (114)

Sol. $\frac{hc}{\lambda} = 13.6z^2 \left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right]$

$$\frac{1242 \times 10^{-9}}{\lambda} = 13.6 \times 9 \times \left[\frac{1}{1^2} - \frac{1}{3^2} \right]$$

$$\lambda = 114 \times 10^{-10} \text{ m}$$

25. A cell, stunted by a 8Ω resistance, is balanced across a potentiometer wire of length 3 m. The balancing length is 2 m when the cell is stunted by 4Ω resistance. The value of interval resistance of the cell will be _____ Ω .

Ans. (8)

Sol. $iR = xl$

$$\Rightarrow \frac{\epsilon}{R_1 + r} R_1 = xl_1$$

$$\Rightarrow \frac{\epsilon}{1 + \frac{r}{R_1}} = xl_1 \quad \& \quad \frac{\epsilon}{1 + \frac{r}{R_2}} = xl_2$$

Dividing the Equation,

$$\frac{1 + \frac{r}{R_2}}{1 + \frac{r}{R_1}} = \frac{l_2}{l_1} = \frac{2}{3} \Rightarrow 3 + \frac{3r}{8} = 2 + \frac{2r}{4} \Rightarrow \frac{r}{2} - \frac{3r}{8} = 1$$

$$\Rightarrow r = 8 \Omega$$

26. The current density in a cylindrical wire of radius 4 mm is $4 \times 10^6 \text{ Am}^{-2}$. The current through the outer portion of the wire between radial distances $\frac{R}{2}$ and R is _____ $\pi \text{ A}$.

Ans. (48)

Sol. $i = J \times \pi \left(R^2 - \frac{R^2}{2} \right)$

$$= 4 \times 10^6 \times \pi \times 16 \times 10^{-6} \times \frac{3}{4}$$

$$= 48\pi \text{ Amp}$$

27. A capacitor of capacitance 50 pF is charged by 100 V source. It is then connected to another uncharged identical capacitor. Electrostatic energy loss in the process is _____ nJ.

Ans. (125)

Sol. Energy loss $H = \frac{1}{2} \frac{C_1 C_2}{C_1 + C_2} (V_1 - V_2)^2$

$$\Rightarrow \frac{1}{2} \frac{C^2}{2C} (V - 0)^2$$

$$\Rightarrow H = \frac{1}{4} CV^2$$

$$\Rightarrow H = \frac{1}{4} \times 50 \times 10^{-12} \times 100 \times 100$$

$$\Rightarrow H = 1.25 \times 10^{-7} \text{ J} = 125 \times 10^{-9} \text{ J}$$

28. The height of a transmitting antenna at the top of a tower is 25 m and that of receiving antenna is, 49 m. The maximum distance between them, for satisfactory communication in Los (Line-Of-Sight) is $K\sqrt{5} \times 10^2$ m. The value of K is _____.

Ans. (192)

Sol. $d = \sqrt{2Rh_1} + \sqrt{2Rh_2}$

$$= \sqrt{2R}(5 + 7)$$

$$= \sqrt{6400 \times 2 \times 10^3} (12)$$

$$= 12\sqrt{64 \times 10^4 \times 20}$$

$$= 12 \times 800 \times 2\sqrt{5} = 24 \times 800\sqrt{5} = 19200\sqrt{5}$$

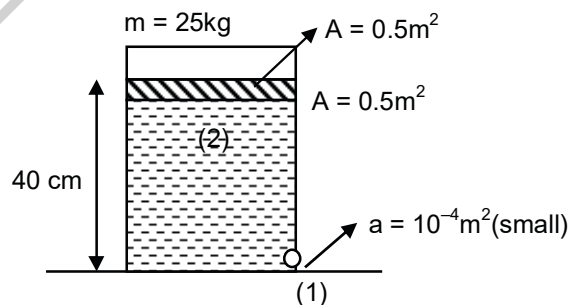
$$= 192\sqrt{5} \times 10^2$$

$$K = 192$$

29. The area of cross-section of a large tank is 0.5 m^2 . It has a narrow opening near the bottom having area of cross-section 1 cm^2 . A load of 25 kg is applied on the water at the top in the tank. Neglecting the speed of water in the tank, the velocity of the water, coming out of the opening at the time when the height of water level in the tank is 40 cm above the bottom, will be _____ cms^{-1} . [Take $g = 10 \text{ ms}^{-2}$]

Ans. (300)

Sol. Apply Bernoulli equation at top and bottom



Points (1) and (2)

$$P_0 + \frac{mg}{A} + 0 + \rho \cdot g \cdot h = P_0 + \frac{1}{2} \rho u^2 + 0$$

$$\frac{250}{5} + 10^3 \times 10 \times 0.4 = \frac{1}{2} \times 10^3 \times u^2$$

$$v = 3 \text{ m/s} = 300 \text{ cm/s}$$

30. A pendulum of length 2 m consists of a wooden bob of mass 50g. A bullet of mass 75 g is fired towards the stationary bob with a speed v . The bullet emerges out of the bob with a speed $\frac{v}{3}$ and the bob just completes the vertical circle. The value of v is _____ ms^{-1} . (if $g = 10 \text{m/s}^2$).

Ans. (10)

Sol. Pendulum length 2m

$$75V = 50\sqrt{5rg} + 75(V/3)$$

$$75\left(\frac{2V}{3}\right) = 50\sqrt{5 \times 2 \times 10}$$

$$75\left(\frac{2V}{3}\right) = 50 \times 10$$

$$V = \frac{500 \times 3}{150} = \frac{500}{50} = 10 \text{m/s}$$

PART : CHEMISTRY

1. Given below are two statements: one is labelled as Assertion(A) and the other is labelled as Reason (R).

Assertion(A): At 10°C , the density of a 5 M solution of KCl [atomic masses of K & Cl are 39 & 35.5 g mol^{-1} respectively], is 'x' g ml^{-1} . The solution is cooled to -21°C . The molality of the solution will remain unchanged.

Reason (R): The molality of a solution does not change with temperature as mass remains unaffected with temperature.

In the light of the above statements, choose the correct answer from the options given below.

- (A) Both (A) and (R) are true and (R) is the correct explanation of (A).
 (B) Both (A) and (R) are true but (R) is not the correct explanation of (A).
 (C) (A) is true but (R) is false.
 (D) (A) is false but (R) is true.

Ans. (A)

Sol. Molality & mass are temperature independent so on changing temperature molality & mass remain unchanged.

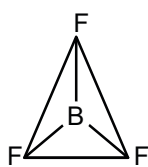
2. Based upon VSEPR theory, match the shape (geometry) of the molecules in list-I with the molecules in List – II and select the most appropriate option.

List – I (Shape)	List – II (Molecules)
(A) T-shaped	(I) XeF_4
(B) Trigonal planar	(II) SF_4
(C) square planner	(III) ClF_3
(D) see-saw	(IV) BF_3

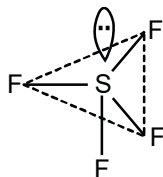
(A) (A) – (I), (B) – (II), (C) – (III), (D) – (IV)
 (B) (A) – (III), (B) – (IV), (C) – (I), (D) – (II)
 (C) (A) – (III), (B) – (IV), (C) – (II), (D) – (I)
 (D) (A) – (IV), (B) – (III), (C) – (I), (D) – (II)

Ans. (B)

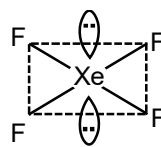
Sol. (I) BF_3	(A) T-shaped
(II) SF_4	(B) Trigonal planar
(III) XeF_4	(C) square planner
(IV) ClF_3	(D) see-saw
(I)	(II)
	(III)
	(IV)



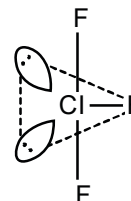
(Trigonal planar)



(See-saw)



(Square planar)



(T-shape)

3. Match List – I with List – II.

List – I

- (A) Spontaneous process
- (B) Process with $\Delta P = 0, \Delta T = 0$
- (C) $\Delta H_{\text{reaction}}$
- (D) Exothermic

List – II

- (I) $\Delta H < 0$
- (II) $\Delta G_{T,P} < 0$
- (III) Isothermal and isobaric process
- (IV) [Bond energies of molecules in reactants] – [Bond energies of product molecules]

Choose the correct answer from the options given below :

- (A) (A) – (III), (B) – (II), (C) – (IV), (D) – (I)
- (B) (A) – (II), (B) – (III), (C) – (IV), (D) – (I)
- (C) (A) – (II), (B) – (III), (C) – (I), (D) – (IV)
- (D) (A) – (II), (B) – (I), (C) – (III), (D) – (IV)

Ans. (B)

- Sol. (I) For spontaneous process $\Rightarrow \Delta G < 0$
 (II) For exothermic process $\Rightarrow \Delta H < 0$
 (III) For isothermal process $\Rightarrow \Delta T < 0$
 For isobaric process $\Rightarrow \Delta P < 0$

4. Match List – I with List – II.

List – I

- (A) Lyophilic colloid
- (B) Emulsion
- (C) Positively charged colloid
- (D) Negatively charged colloid

List – II

- (I) Liquid-liquid colloid
- (II) Protective colloid
- (III) $\text{FeCl}_3 + \text{NaOH}$
- (IV) $\text{FeCl}_3 + \text{hot water}$

Choose the correct answer from the options given below :

- (A) (A) – (II), (B) – (I), (C) – (IV), (D) – (III)
- (B) (A) – (III), (B) – (I), (C) – (IV), (D) – (II)
- (C) (A) – (II), (B) – (I), (C) – (III), (D) – (IV)

(D) (A) – (III), (B) – (II), (C) – (I), (D) – (IV)

Ans. (A)

Sol. (A) Lyophilic colloid \Rightarrow Protective colloid

(B) Emulsion \Rightarrow Liquid-Liquid colloid

(C) Positively charged colloid \Rightarrow FeCl_3 + hot water

(D) Negatively charged colloid \Rightarrow FeCl_3 + NaOH

5. Given below are two statements: one is labelled as Assertion(A) and the other is labelled as Reason (R).

Assertion(A): The ionic radii of O^{2-} and Mg^{2+} are same.

Reason (R): Both O^{2-} and Mg^{2+} are isoelectronic species.

In the light of the above statements, choose the correct answer from the options given below.

(A) Both (A) and (R) are true and (R) is the correct explanation of (A).

(B) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(C) (A) is true but (R) is false.

(D) (A) is false but (R) is true.

Ans. (D)

Sol.	Ion	O^{2-}	Mg^{2+}
	No. of e^-	10	10
	Z	8	12

Order of ionic size $\Rightarrow \text{O}^{2-} > \text{Mg}^{2+}$

6. Match List – I with List – II.

List – I	List – II
(A) Concentration of gold ore	(I) Aniline
(B) Leaching of alumina	(II) NaOH
(C) Froth stabilizer	(III) SO_2
(D) Blister copper	(IV) NaCN

Choose the correct answer from the options given below :

(A) (A) – (IV), (B) – (III), (C) – (II), (D) – (I)

(B) (A) – (IV), (B) – (II), (C) – (I), (D) – (III)

(C) (A) – (III), (B) – (II), (C) – (I), (D) – (IV)

(D) (A) – (II), (B) – (IV), (C) – (III), (D) – (I)

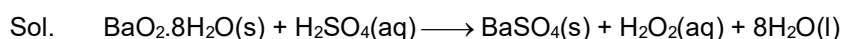
Ans. (B)

Sol. Theory based

7. Addition of H_2SO_4 to BaO_2 produces:

- (A) BaO , SO_2 and H_2O
- (B) BaHSO_4 and O_2
- (C) BaSO_4 , H_2 and O_2
- (D) BaSO_4 and H_2O_2

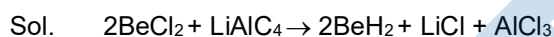
Ans. (D)



8. BeCl_2 reacts with LiAlH_4 to give:

- (A) $\text{Be} + \text{Li}[\text{AlCl}_4] + \text{H}_2$
- (B) $\text{Be} + \text{AlH}_3 + \text{LiCl} + \text{HCl}$
- (C) $\text{BeH}_2 + \text{LiCl} + \text{AlCl}_3$
- (D) $\text{BeH}_2 + \text{Li}[\text{AlCl}_4]$

Ans. (C)



9. Match List – I with List – II.

- | List – I
(Si-Compounds) | List – II
(Si-Polymeric/Other Products) |
|---|--|
| (A) $(\text{CH}_3)_4\text{Si}$ | (I) Chain Silicone |
| (B) $(\text{CH}_3)\text{Si}(\text{OH})_3$ | (II) Dimeric Silicon |
| (C) $(\text{CH}_3)_2\text{Si}(\text{OH})_2$ | (III) Silane |
| (D) $(\text{CH}_3)_3\text{Si}(\text{OH})$ | (IV) 2D - Silicone |

Choose the correct answer from the options given below :

- (A) (A) – (III), (B) – (II), (C) – (I), (D) – (IV)
- (B) (A) – (IV), (B) – (I), (C) – (II), (D) – (III)
- (C) (A) – (II), (B) – (I), (C) – (IV), (D) – (III)
- (D) (A) – (III), (B) – (IV), (C) – (I), (D) – (II)

Ans. (D)

- Sol.
- | | |
|--|------------------|
| (i) $\text{Si}(\text{CH}_3)_4$ | Silane |
| (ii) $\text{Si}(\text{CH}_3)_2(\text{OH})_2$ | Chain silicone |
| (iii) $\text{Si}(\text{CH}_3)(\text{OH})_3$ | 2D silicone |
| (iv) $\text{Si}(\text{CH}_3)_3(\text{OH})$ | Dimeric silicone |

10. Heating white phosphorus with conc. NaOH solution gives mainly:

- (A) Na_3P and H_2O

- (B) H_3PO and NaH
 (C) $\text{P}(\text{OH})_3$ and NaH_2PO_4
 (D) PH_3 and NaH_2PO_2

Ans. (D)

Sol. $\text{P}_4(\text{white}) + \text{NaOH} \longrightarrow \text{PH}_3 + \text{NaH}_2\text{PO}_2$

11. Which of the following have maximum stabilization due to crystal field ?

- (A) $[\text{Ti}(\text{H}_2\text{O})_6]^{3+}$
 (B) $[\text{Co}(\text{H}_2\text{O})_6]^{2+}$
 (C) $[\text{Co}(\text{CN})_6]^{3-}$
 (D) $[\text{Cu}(\text{NH}_3)_4]^{2+}$

Ans. (C)

Sol.

Complex	Hybridisation
---------	---------------

- | | |
|--|-----------|
| (1) $[\text{Ti}(\text{H}_2\text{O})_6]^{3+}$ | d^2sp^3 |
| (2) $[\text{Co}(\text{H}_2\text{O})_6]^{2+}$ | sp^3d^2 |
| (3) $[\text{Co}(\text{CN})_6]^{3-}$ | d^2sp^3 |
| (4) $[\text{Cu}(\text{NH}_3)_4]^{2+}$ | dsp^2 |

$$\Delta_{sp} = 1.3 \Delta_0$$

12. Given below are two statements:

Statement I : Classical smog occurs in cool humid climate. It is a reducing mixture of smoke, fog and sulphur dioxide.

Statement II : Photochemical smog has components, ozone, nitric oxide, acrolein, formaldehyde, PAN etc.

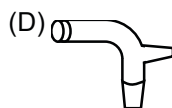
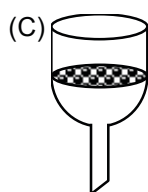
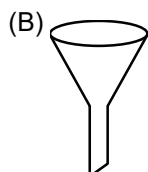
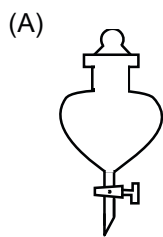
In the light of the above statements, choose the most appropriate answer from the options given below.

- (A) Both Statement I and Statement II are correct.
 (B) Both Statement I and Statement II are incorrect.
 (C) Statement I is correct but Statement II is incorrect.
 (D) Statement I is incorrect but Statement II is correct.

Ans. (A)

Sol. It is fact.

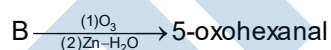
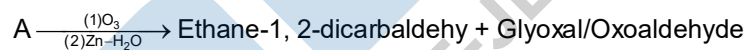
13. Which of the following is structure of a separating ?



Ans. (A)

Sol. It is fact.

14. 'A' and 'B' respectively are:



(A) 1-methylcyclohex-1,3-diene & cyclopentane.

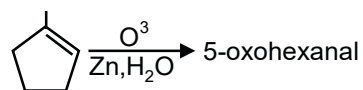
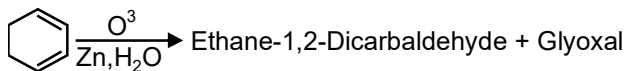
(B) Cyclohex-1,3-diene & cyclopentane.

(C) 1-methylcyclohex-1,4-diene & 1-methylcyclopent-1-ene

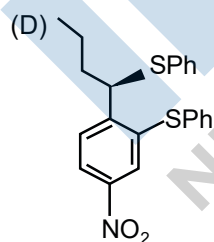
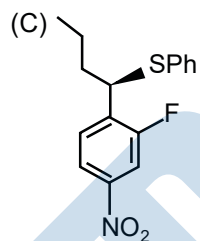
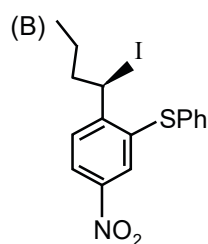
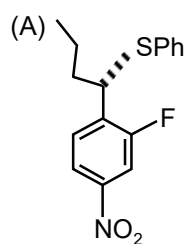
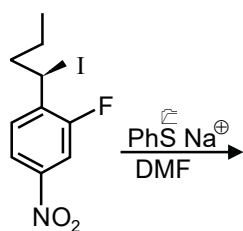
(D) Cyclohex-1, 3-diene & 1-methylcyclopent-1-ene

Ans. (D)

Sol.

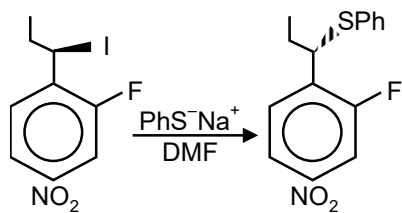


15. The major product of the following reaction is :

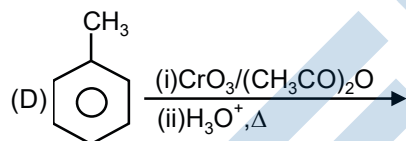
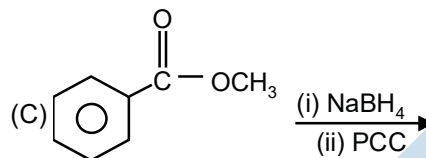
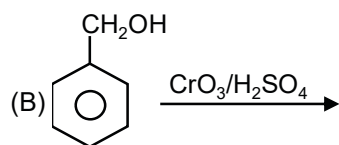
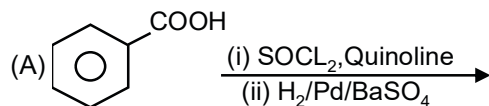


Ans. (A)

Sol. In SN^2 reaction on version take place



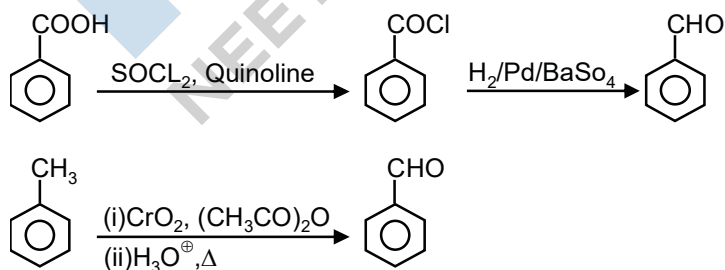
16. Which of the following reactions will yield benzaldehyde as a product ?



- (A) (B) and (C)
- (B) (C) and (D)
- (C) (A) and (D)
- (D) (A) and (C)

Ans. (C)

Sol.



17. Given below are two statement:

Statement – I : In Hofmann degradation reaction, the migration of only an alkyl group takes place from carbonyl carbon of the amide to the nitrogen atom.

Statement – II: The group is migrate in Hofmann degradation reaction to electron deficient atom.

In the light of the above statement, choose the most appropriate answer from the options given below :

- (A) Both Statement I and Statement II are correct.
 (B) Both Statement I and Statement II are incorrect.
 (C) Statement I is correct but Statement II is incorrect.
 (D) Statement I is incorrect but Statement II is correct.

Ans. (D) (BONUS)

Sol. Both Statement I and statement II are incorrect statement is given in the frame of Hofmaan degradation reaction but there is no such methylmygration in Hofmaan degradation.

18. Match List – I with List – II.

List – I (Polymer)	List – II (Used in)
(A) Bakelite	(I) Radio and television cabinets
(B) Glyptal	(II) Electrical switches
(C) PVC	(III) Paints and Lacquers
(D) Polystyrene	(IV) Water pipes

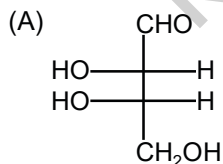
Choose the correct answer from the options given below :

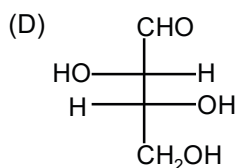
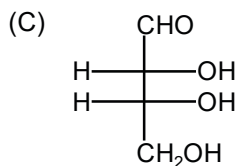
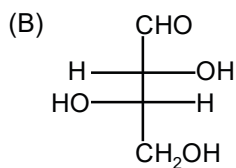
- (A) (A) – (II), (B) – (III), (C) – (IV), (D) – (I)
 (B) (A) – (I), (B) – (II), (C) – (III), (D) – (IV)
 (C) (A) – (IV), (B) – (III), (C) – (II), (D) – (I)
 (D) (A) – (II), (B) – (III), (C) – (I), (D) – (IV)

Ans. (A)

Sol. It is fact.

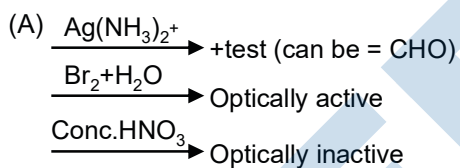
19. L-isomer of a compound 'A' ($C_4H_8O_4$) gives a positive test with $[Ag(NH_3)_2]^+$. Treatment of 'A' with acetic anhydride yields triacetate derivative. Compound 'A' produces an optically active compound (B) and an optically inactive compound (C) on treatment with bromine water and HNO_3 respectively. Compound (A) is:





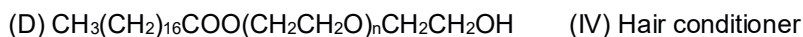
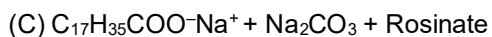
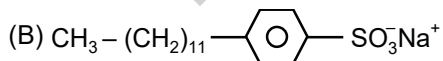
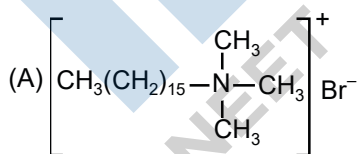
Ans. (A)

Sol. $\text{C}_6\text{H}_8\text{O}_6 \rightarrow \text{DU} = 1$



20. Match List – I with List II.

List – I



List – II

(I) Dishwashing powder

(II) Toothpaste

(III) Laundry soap

(IV) Hair conditioner

Choose the correct answer from the options given below :

(A) (A) – (III), (B) – (II), (C) – (IV), (D) – (I)

(B) (A) – (IV), (B) – (II), (C) – (III), (D) – (I)

(C) (A) – (IV), (B) – (III), (C) – (II), (D) – (I)

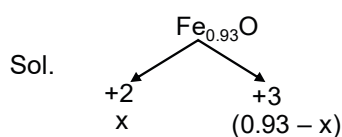
(D) (A) – (III), (B) – (IV), (C) – (I), (D) – (I)

Ans. (B)

Sol. It is fact.

21. Metal deficiency defect is shown by $\text{Fe}_{0.93}\text{O}$. In the crystal, some Fe^{2+} cations are missing and loss of positive charge is compensated by the presence of Fe^{3+} ions. The percentage of Fe^{2+} ions in the $\text{Fe}_{0.93}\text{O}$ crystal is _____. (Nearest integer)

Ans. (85)



$$2x + 3(0.93 - x) = 2$$

$$2x + 0.93 \times 3 - 3x = 2$$

$$2.79 - 2 = x$$

$$\% \text{ of } \text{Fe}^{2+} = \frac{0.79}{0.93} \times 100 = 84.94\%$$

Ans. = 85

22. If the uncertainty in velocity and position of a minute particle in space are, $2.3 \times 10^{-26} \text{ (ms}^{-1}\text{)}$ and 10^{-7} (m) respectively. The mass of the particle in g is _____. (Nearest integer)

(Given : $h = 6.626 \times 10^{-34} \text{ Js}$)

Ans. (22)

Sol. According to Heisenberg uncertainty Principle

$$\Delta x \times \Delta P \geq \frac{h}{4\pi}$$

$$\Rightarrow 10^{-7} \times m \cdot \Delta V = \frac{6.62 \times 10^{-34}}{4 \times 3.14}$$

$$\Rightarrow 10^{-7} \times m \times 2.4 \times 10^{-24} = \frac{6.62 \times 10^{-34}}{4 \times 3.14}$$

$$M = 0.2196 \times 10^{-3} \text{ Kg}$$

$$= 21.96 \times 10^{-5} \text{ Kg} \quad \text{Ans.} = 22$$

23. 2 g of none-volatile non-electrolyte solute is dissolve in 200g of two different solvents A and B whose ebullioscopic constants are in the ratio of 1 : 8. The elevation in boiling points of A and B are in the ratio $\frac{x}{y}$ (x : y). The value of y is _____. (Nearest Integer)

Ans (8)

Sol. $\Delta T_b = K_b \times m$

$$\frac{(\Delta T_b)_I}{(\Delta T_b)_{II}} = \frac{(K_b)_I}{(K_b)_{II}} = \frac{1}{8} = \frac{x}{y}$$

So y = 8

24. $2\text{NOCl(g)} \rightleftharpoons 2\text{NO(g)} + \text{Cl}_2\text{(g)}$

In an experiment, 2.0 moles of NOCl was placed in a one-litre flask and the concentration of NO after equilibrium established, was found to be 0.4 mol/L. The equilibrium constant at 30°C is _____ $\times 10^{-4}$.

Ans. (125)

Sol. $2\text{NOCl(g)} \rightleftharpoons 2\text{NO} + \text{Cl}_2$

Initially	2 mole	0	0
At equilibrium	(2 - 0.4)	0.4	0.2
	= 1.6		

$$K_c = \frac{(\text{NO})^2(\text{Cl}_2)}{(\text{NOCl})^2} = \frac{(0.4)^2(0.2)}{(1.6)^2} = 0.0125 = 125 \times 10^{-4}$$

Ans = 125

25. The limiting molar conductivities of NaI, NaNO₃ and AgNO₃ are 12.7, 12.0 and 13.3 mS m² mol⁻¹, respectively (all at 25°C). The limiting molar conductivity of AgI at this temperature is _____ mS m² mol⁻¹.

Ans. (14)

Sol. Given

$$\lambda_m^\infty(\text{NaI}) = 12.7 \text{ S m}^2 \text{ mole}^{-2}$$

$$\lambda_m^\infty(\text{AgNO}_3) = 13.3 \text{ S m}^2 \text{ mole}^{-2}$$

$$\lambda_m^\infty(\text{NaNO}_3) = 12 \text{ S m}^2 \text{ mole}^{-2}$$

$$\lambda_m^\infty(\text{AgI}) = \lambda_m^\infty(\text{AgNO}_3) + \lambda_m^\infty(\text{NaI}) - \lambda_m^\infty(\text{NaNO}_3)$$

$$= 13.3 + 12.7 - 12$$

$$= 14 \text{ S m}^2 \text{ mole}^{-2}$$

26. The rate constant for a first order reaction is given by the equation :

$$\ln k = 33.24 - \frac{2.0 \times 10^4 \text{ K}}{T}$$

The Activation energy for the reaction is given by _____ kJ mol^{-1} . (In Nearest integer)

(Given : $R = 8.3 \text{ J K}^{-1} \text{ mol}^{-1}$)

Ans. (166)

Sol. $\ln k = 33.24 - \frac{2.0 \times 10^4}{T} \text{K}$

$$K = Ae^{\frac{E_a}{RT}}$$

$$\ln k = \ln A - \frac{E_a}{10000 \times R} \times \frac{10^4}{T}$$

$$\text{so} \left(\frac{E_a}{10^4 \times R} \right) = 2$$

$$E_a = 2 \times 8.3 \times 10^4$$

$$E_a = 166 \text{ kJ}$$

27. The number of statement(s) correct from the following for Copper (at. No. 29) is/are _____.

(A) Cu (II) complexes are always paramagnetic

(B) Cu (I) complexes are generally colourless

(C) Cu (I) is easily oxidized

(D) In Fehling solution, the active reagent has Cu(I)

Ans. (3)

Sol. (A) Cu^{2+} has $3d^9$ Electronic configuration so its complexes are paramagnetic

(B) Cu^{+1} has $3d^{10}$ Electronic configuration so its complexes are colourless

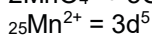
(C) Fehling solution contain aqueous solution of CuSO_4

All three statement are correct.

28. Acidified potassium permanganate solution oxidises oxalic acid. The spin-only magnetic moment of the manganese product formed from the above reaction is _____ B.M.

(Nearest integer)

Ans. (6)



No. of unpaired electron = 5

$$\mu(\text{spin only}) = \sqrt{n(n+2)} \text{BM}$$

$$= \sqrt{5(5+2)} = \sqrt{35}$$

$$= 5.916 \approx 6$$

29. Two elements A and B which form 0.15 moles of A_2B and AB_3 type compounds. If both A_2B and AB_3 weigh equally, then the atomic weight of A is _____ times of atomic weight of B.

Ans. (2)

Sol. Let molar mass of A is a

B is b

$$\text{Again} \Rightarrow 0.15 [2a + b] = 0.15 [a + 3b]$$

$$a = 2b \Rightarrow \left(\frac{a}{b}\right) = \frac{2}{1}$$

30. Total number of possible stereoisomers of dimethyl cyclopentane is _____.

Ans. (6)

MENIIT
NEET | IIT-JEE | FOUNDATION

Part : Mathematics

SECTION-A

1. The area of the polygon, whose vertices are the non-real roots of the equation $\bar{z} = iz^2$ is :

- (A) $\frac{3\sqrt{3}}{4}$ (B) $\frac{3\sqrt{3}}{2}$ (C) $\frac{3}{2}$ (D) $\frac{3}{4}$

Ans. (A)

Sol. \Rightarrow Let $z = x + iy$, $x, y \in \mathbb{R}$

Now $\bar{z} = iz^2$

then $x - iy = i(x^2 - y^2 + 2xyi)$

$x - iy = i(x^2 - y^2) - 2xy$

$\Rightarrow x = -2xy$ & $-y = x^2 - y^2$

$\Rightarrow x(1 + 2y) = 0$

$x = 0$ or $y = -\frac{1}{2}$

Put $x = 0$ in $-y = x^2 - y^2$

We get $y = y^2$

$\Rightarrow y = 0, 1$

Similarly

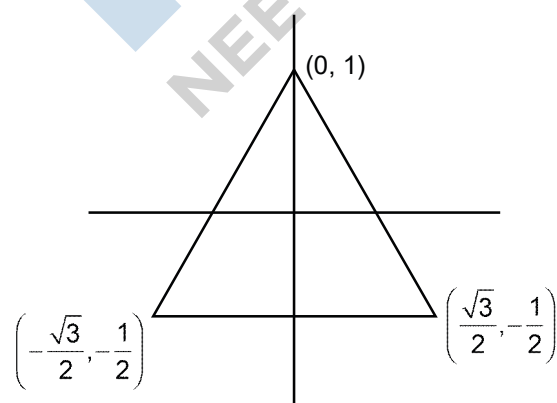
Put $y = -\frac{1}{2}$ in $-y = x^2 - y^2$

$\Rightarrow \frac{1}{2} = x^2 - \frac{1}{4}$

$\Rightarrow x^2 = \frac{3}{4}$

$x = \pm \frac{\sqrt{3}}{2}$

$z = \left(0, i, \frac{\sqrt{3}}{2} - \frac{1}{2}i, -\frac{\sqrt{3}}{2} - \frac{1}{2}i \right)$



$$\begin{aligned} \text{Area} &= \frac{1}{2} \cdot (\sqrt{3}) \left(\frac{3}{2}\right) \\ &= \frac{3\sqrt{3}}{4} \end{aligned}$$

2. Let the system of linear equation $x + 2y + z = 2$, $\alpha x = 3y - z = \alpha$, $-\alpha x + y + 2z = -\alpha$ be inconsistent Then α is equal to :

- (A) $\frac{5}{2}$ (B) $-\frac{5}{2}$ (C) $\frac{7}{2}$ (D) $-\frac{7}{2}$

Ans. (D)

Sol. $\Delta = \begin{vmatrix} 1 & 2 & 1 \\ 2 & 3 & -1 \\ -2 & 1 & 2 \end{vmatrix}$

$$\begin{aligned} &= (6 + y) - 2(2\alpha - \alpha) + 1(\alpha + 3\alpha) \\ &= 7 - 2\alpha + 4\alpha \\ &= 7 + 2\alpha \end{aligned}$$

$$\Delta = 0 \Rightarrow \alpha = -\frac{7}{2}$$

$$\Delta_1 = \begin{vmatrix} 2 & 2 & 1 \\ \alpha & 3 & -1 \\ -\alpha & 1 & 2 \end{vmatrix}$$

$$= 14 + 2\alpha$$

$$\alpha = -x_2 = 7$$

$$\Delta_1 \neq 0$$

3. If $x = \sum_{n=0}^{\infty} a^n, y = \sum_{n=0}^{\infty} b^n, z = \sum_{n=0}^{\infty} c^n$, where a, b, c are in A.P. and $|a| < 1, |b| < 1, |c| < 1, abc \neq 0$, then

- (A) x, y, z are in A.P.
 (B) x, y, z are in G.P.
 (C) $\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ are in A.P.
 (D) $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1 - (a + b + c)$

Ans. (C)

Sol. $x = 1 + a + a^2 = \dots\dots\dots$

$$x = \frac{1}{1-a} \Rightarrow a = 1 - \frac{1}{x}$$

$$y = \frac{1}{1-b} \Rightarrow b = 1 - \frac{1}{y}$$

$$z = \frac{1}{1-c} \Rightarrow c = 1 - \frac{1}{z}$$

a, b, c are in A.P.

$$\Rightarrow 1 - \frac{1}{x}, 1 - \frac{1}{y}, 1 - \frac{1}{z} \text{ are in A.P.}$$

$$\Rightarrow -\frac{1}{x}, -\frac{1}{y}, -\frac{1}{z} \text{ are in A.P.}$$

$$\Rightarrow \frac{1}{x}, \frac{1}{y}, \frac{1}{z} \text{ are in A.P.}$$

4. Let $\frac{dy}{dx} = \frac{ax - by + a}{bx + cy + a}$, where a, b, c are constants, represent a circle passing through the point (2, 5). Then the shortest distance of the point (11, 6) from this circle is :
- (A) 10 (B) 8 (C) 7 (D) 5

Ans. (B)

Sol. Let equation of circle is

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{-(2x + 2g)}{(2y + 2f)}$$

Comparing with $\frac{dy}{dx} = \frac{ax - by + a}{bx + cy + a}$

$$\Rightarrow b = 0, a = -2, c = 2$$

$$\Rightarrow -2g = -2 \Rightarrow g = 1 \quad 2f = -2$$

$$F = -1$$

Now circle will be

$$x^2 + y^2 + 2x + 2y + c = 0$$

its passes through (2, 5)

which will give $c = -23$

so circle will be $x^2 + y^2 + 2x - 2y - 23 = 0$

centre C = (-1, 1)

and radius 5

Now P is (11, 6)

So minimum distance of P from circle will be

$$= \sqrt{(11+1)^2 + (6-1)^2} - 5$$

$$= 13 - 5$$

$$= 8$$

5. Let a be an integer such that $\lim_{x \rightarrow 7} \frac{18 - [1-x]}{[x] - 3a}$ exists, where [t] is greatest integer $\leq t$. Then a is equal

to :

- (A) -6 (B) -2 (C) 2 (D) 6

Ans. (A)

Sol. $\lim_{x \rightarrow 7} \frac{18 - [1-x]}{[x] - 3a}$

L.H.L. $\lim_{x \rightarrow 7} \frac{18 - [1-x]}{[x] - 3a}$

$$= \frac{18 - (-6)}{6 - 3a}$$

$$= \frac{24}{6 - 3a}$$

R.H.L. $\lim_{x \rightarrow 7+} \frac{18 - [1-x]}{[x] - 3a}$

$$= \frac{18 - (-7)}{7 - 3a}$$

$$= \frac{25}{7 - 3a}$$

Now L.H.L. = R.H.L.

$$= \frac{24}{6 - 3a} = \frac{25}{7 - 3a}$$

$$\Rightarrow 168 - 72a = 150 - 75a$$

$$\Rightarrow 18 = -3a$$

$$\Rightarrow a = -6$$

6. The number of distinct real roots of $x^4 - 4x + 1 = 0$ is :

- (A) 4 (B) 2 (C) 1 (D) 0

Ans. (B)

Sol. Let $f(x) = x^4 - 4x + 1$

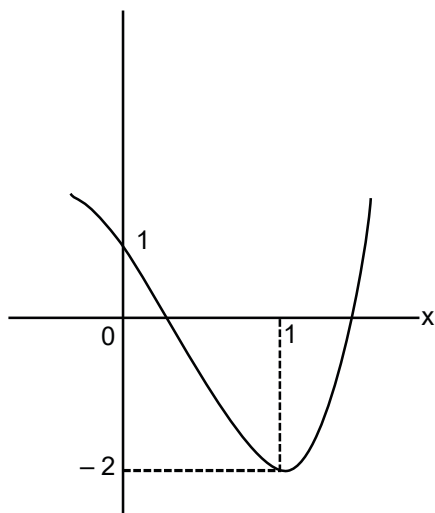
$$f'(x) = 4x^3 - 4$$

$$f'(x) = 0 \Rightarrow x = 1$$

$x = 1$ is point of minima.

$$f(1) = -2$$

$$f(0) = 1$$



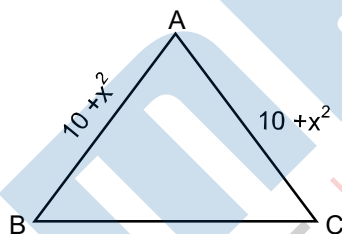
Hence 2 solutions.

7. The lengths of the sides of a triangle are $10 + x^2$, $10 + x^2$ and $20 - 2x^2$. If for $x = k$, the area of the triangle is maximum, then $3k^2$ is equal to :

- (A) 5 (B) 8 (C) 10 (D) 12

Ans. (C)

Sol.



$$a = 20 - 2x^2, b = 10 + x^2, c = 10 + x^2$$

$$= \frac{a+b+c}{2}$$

$$= 20$$

$$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{20(2x^2)(10 - x^2)(10 - x^2)}$$

$$= 2\sqrt{10} \sqrt{x^2(10 - x^2)^2}$$

$$= 2\sqrt{10}|x(10 - x^2)|$$

$$= 2\sqrt{10}|(10x - x^3)|$$

$$S = 10x - x^3$$

$$\frac{ds}{dx} = 10 - 3x^2$$

$$\frac{ds}{dx} = 0 \Rightarrow x^2 = \frac{10}{3}$$

$$3x^2 = 10$$

8. If $\cos^{-1}\left(\frac{y}{2}\right) = \log_e\left(\frac{x}{5}\right)^5$, $|y| < 2$, then :

- (A) $x^2 y'' + xy' - 25y = 0$
- (B) $x^2 y'' - xy' - 25y = 0$
- (C) $x^2 y'' - xy' + 25y = 0$
- (D) $x^2 y'' + xy' + 25y = 0$

Ans. (D)

Sol. $\cos^{-1}\left(\frac{y}{2}\right) = \log_e\left(\frac{x}{5}\right)^5$

$$\cos^{-1}\left(\frac{y}{2}\right) = 5\log_e\left(\frac{x}{5}\right)$$

$$\frac{-1}{\sqrt{1-\frac{y^2}{4}}} \cdot \frac{y'}{2} = 5 \cdot \frac{1}{x} \times \frac{1}{5}$$

$$\Rightarrow \frac{-y'}{\sqrt{4-y^2}} = \frac{5}{x}$$

$$-xy' = 5\sqrt{4-y^2}$$

$$-xy'' - y' = 5 \cdot \frac{1}{2\sqrt{4-y^2}} (-2yy')$$

$$\Rightarrow xy'' + y' = \frac{5y'y}{\sqrt{4-y^2}}$$

$$xy'' + y' = 5 \cdot \left(\frac{-5}{x}\right) y$$

$$x^2 y'' + xy' = -25y$$

9. $\int \frac{(x^2 + 1)e^x}{(x + 1)^2} dx = f(x)e^x + C$, Where C is a constant, then $\frac{d^3f}{dx^3}$ at $x = 1$ is equal to :

- (A) $-\frac{3}{4}$ (B) $\frac{3}{4}$ (C) $-\frac{3}{2}$ (D) $\frac{3}{2}$

Ans. (B)

Sol.
$$\int \left(\frac{x^2 + 1}{(x + 1)^2} \right) e^x \cdot dx$$

$$= \int \left(\frac{x^2 - 1 + 2}{(x + 1)^2} \right) e^x dx$$

$$= \int \left(\frac{x - 1}{x + 1} + \frac{2}{(x + 1)} \right) e^x dx$$

$$= \int (f(x) + f'(x)) e^x dx$$

$$= f(x) e^x + c$$

Where $f(x) = \frac{x - 1}{x + 1}$

$$f'(x) = \frac{2}{(x + 1)^2}$$

$$f''(x) = \frac{-4}{(x + 1)^3}$$

$$= \frac{12}{(x + 1)^4}$$

$$f''(1) = \frac{12}{16}$$

$$= \frac{3}{4}$$

10. The value of the integral $\int_{-2}^2 \frac{|x^3 + x|}{(e^{x|x|} + 1)} dx$ is equal to :

- (A) $5e^2$ (B) $3e^{-2}$ (C) 4 (D) 6

Ans. (D)

Sol. $f(x) = \frac{|x^3 + x|}{(e^{x|x|} + 1)} dx$

$$\begin{aligned} \int_{-2}^2 f(x) dx &= \int_0^2 (f(x) + f(-x)) dx \\ &= \int_0^2 \left(\frac{|x^3 + x|}{(e^{x|x|} + 1)} + \frac{|-x^3 - x|}{(e^{-x|x|} + 1)} \right) dx \\ &= \int_0^2 \left(\frac{|x^3 + x|}{(e^{x|x|} + 1)} + \frac{|x^3 + x|}{(e^{-x|x|} + 1)} \right) dx \\ &= \int_0^2 \left(\frac{x^3 + x}{(e^{x^2} + 1)} + \frac{x^3 + x}{(e^{-x^2} + 1)} \right) dx \\ I &= \int_0^2 \left(\frac{x^3 + x}{1 + e^{x^2}} + \frac{e^{x^2}(x^3 + x)}{1 + e^{x^2}} \right) dx \\ &= \int_0^2 (x^3 + x) dx \\ &= \left[\frac{x^4}{4} + \frac{x^2}{2} \right]_0^2 \\ &= 4 + 2 = 6 \end{aligned}$$

11. If $\frac{dy}{dx} + \frac{2^{x-y}(2^y - 1)}{2^x - 1} = 0, x, y > 0, y(1) = 1$, then $y(2)$ is equal to :

- (A) $2 + \log_2 3$ (B) $2 + \log_2 2$ (C) $2 - \log_2 2$ (D) $2 - \log_2 3$

Ans. (D)

Sol. $\frac{dy}{dx} + \frac{2^{x-y}(2^y - 1)}{2^x - 1} = 0, x, y > 0, y(1) = 1, y(2) = ?$

$$\frac{dy}{dx} + \frac{2^x(2^y - 1)}{2^y(2^x - 1)}$$

$$\int \frac{2^y}{2^y - 1} dy = - \int \frac{2^x}{2^x - 1} dx$$

$$\frac{1}{\ln 2} \int \frac{2^y \ln 2}{2^y - 1} dy = - \frac{1}{\ln 2} \int \frac{2^x \ln 2}{2^x - 1} dx$$

$$\frac{1}{\ln 2} \ln|2^y - 1| = \frac{-1}{\ln 2} \ln|2^x - 1| + C$$

At $x = 1, y = 1$

Putting this values in above relation we get $C = 0$

$$\ln|2^y - 1| + \ln|2^x - 1| = 0$$

$$(2^x - 1)(2^y - 1) = 1$$

$$2^y - 1 = \frac{1}{2^x - 1}$$

At $x = 2$

$$2^y - 1 = \frac{1}{2^2 - 1} = \frac{1}{3}$$

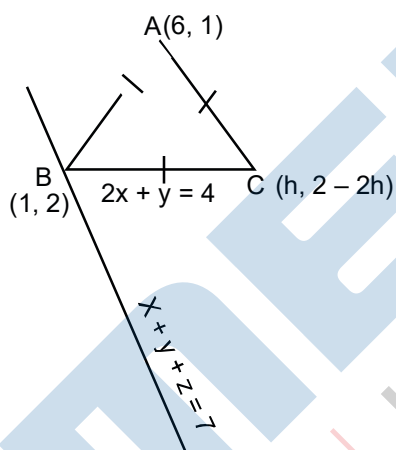
$$2^y = \frac{4}{3} \Rightarrow y = \log_2 \frac{4}{3} = \log_2 4 - \log_2 3 = 2 - \log_2 3$$

12. In an isosceles triangle ABC, the vertex A is (6, 1) and the equation of the base BC is $2x + y = 4$. Let the point B lie on the line $x + 3y = 7$. If (α, β) is the centroid ΔABC , then $15(\alpha + \beta)$ is equal to :

- (A) 39 (B) 41 (C) 51 (D) 63

Ans. (C)

Sol.



Point B (1, 2)

Now let be $(h, 4 - 2h)$

(As C lies on $2x + y = 4$)

$\therefore \Delta$ is isosceles with base BC

$\therefore AB = AC$

$$\sqrt{25 + 1} = \sqrt{(6 - h)^2 + (2h - 3)^2}$$

$$\sqrt{26} = \sqrt{36 + h^2 - 12h + 4h^2 + 9 - 12h}$$

$$26 = 5h^2 - 24h + 45 \Rightarrow 5h^2 - 24h + 19 = 0$$

$$\Rightarrow 5h^2 - 5h - 19h + 19 = 0$$

$$h = \frac{19}{5} \text{ or } h = 1$$

Thus C $\left(\frac{19}{5}, \frac{-18}{5}\right)$

$$\text{Centroid} \left(\frac{6+1+\frac{19}{5}}{3}, \frac{1+2-\frac{18}{5}}{3} \right)$$

$$\left(\frac{35+19}{15}, \frac{15-18}{15} \right)$$

$$\left(\frac{54}{15}, \frac{-3}{15} \right)$$

$$\alpha = \frac{54}{15}; \beta = \frac{-3}{15}$$

$$15(\alpha + \beta) = 51$$

13. Let the eccentricity of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a > b$, be $\frac{1}{4}$. If this ellipse passes through the point $\left(-4, \sqrt{\frac{2}{5}}\right), 3$, then $a^2 + b^2$ is equal to:
- (A) 29 (B) 31 (C) 32 (D) 34

Ans. (B)

Sol. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a > b$

$$e^2 = 1 - \frac{b^2}{a^2}$$

$$\frac{1}{16} = 1 - \frac{b^2}{a^2}$$

$$\frac{b^2}{a^2} = 1 - \frac{1}{16} = \frac{15}{16} \Rightarrow b^2 = \frac{15}{16}a^2$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{16 \cdot \frac{2}{5}}{a^2} + \frac{9}{b^2} = 1$$

$$\frac{32}{5a^2} + \frac{9}{15a^2} = 1$$

$$\frac{80}{5a^2} = 1$$

$$16 = a^2$$

$$b^2 = 15$$

14. If two straight lines whose direction cosines are given by the relations $l + m - n = 0$, $3l^2 + m^2 + cnl = 0$ are parallel, then the positive value of c is :
 (A) 6 (B) 4 (C) 3 (D) 2

Ans. (A)

Sol. $l + m - n = 0$
 $3l^2 + m^2 + cl(l + m) = 0$
 $n = l + m$
 $3l^2 + m^2 + cl^2 + clm = 0$
 $(3 + c)l^2 + clm + m^2 = 0$

$$(3 + c)\left(\frac{l}{m}\right)^2 + c\left(\frac{l}{m}\right) + 1 = 0 \dots (1)$$

\therefore lines are parallel.

Roots of (1) must be equal

$$\Rightarrow D = 0$$

$$c^2 - 4(3 + c) = 0$$

$$c^2 - 4c - 12 = 0$$

$$(c - 6)(c + 2) = 0$$

$$c = 6 \text{ or } c = -2$$

+ve value of $c = 6$

15. Let $\vec{a} = \hat{i} + \hat{j} - \hat{k}$ and $\vec{c} = 2\hat{i} - 3\hat{j} + 2\hat{k}$. Then the number of vectors \vec{b} such that $\vec{b} \times \vec{c} = \vec{a}$ and $|\vec{b}| \in \{1, 2, \dots, 10\}$ is:
 (A) 0 (B) 1 (C) 2 (D) 3

Ans. (A)

Sol. $\vec{a} = \hat{i} + \hat{j} - \hat{k}$

$$\vec{c} = 2\hat{i} - 3\hat{j} + 2\hat{k}$$

$$\vec{b} \times \vec{c} = \vec{a}$$

$$|\vec{b}| \in \{1, 2, \dots, 10\}$$

$$\therefore \vec{b} \times \vec{c} = \vec{a}$$

$\Rightarrow \vec{a}$ is perpendicular to \vec{b} as well as \vec{a} is perpendicular to \vec{c}

Now $\vec{a} \cdot \vec{c} = 2 - 3 - 2 = -3 \neq 0$

This $\vec{b} \times \vec{c} = \vec{a}$ is not possible.

No. of vectors $\vec{b} = 0$

16. Five numbers x_1, x_2, x_3, x_4, x_5 are randomly selected from the numbers 1, 2, 3, ..., 18 and are arranged in the increasing order ($x_1 < x_2 < x_3 < x_4 < x_5$). The probability that $x_2 = 7$ and $x_4 = 11$ is :

- (A) $\frac{1}{136}$ (B) $\frac{1}{72}$ (C) $\frac{1}{68}$ (D) $\frac{1}{34}$

Ans. (C)

Sol. No. of ways to select and arrange x_1, x_2, x_3, x_4, x_5 from 1, 2, 3, 18

$$n(s) = {}^{18}C_5$$

$x_1(x_2) x_3(x_4) x_5$

7 11

$$n(E) = {}^6C_1 \times {}^3C_1 \times {}^7C_1$$

$$P(E) = \frac{6 \times 3 \times 7}{{}^{18}C_5}$$

$$\frac{1}{17 \times 4} = \frac{1}{68}$$

17. Let X be a random variable having binomial distribution B(7, p). If $P(X = 3) = 5P(X = 4)$, then the sum of the mean and the variance of X is :

- (A) $\frac{105}{16}$ (B) $\frac{7}{16}$ (C) $\frac{77}{36}$ (D) $\frac{49}{16}$

Ans. (C)

Sol. B(7, p)

$$N = 7 \quad p = p$$

given

$$P(x = 3) = 5P(x = 4)$$

$${}^7C_3 \times p^3 (1 - p)^4 = 5 \cdot {}^7C_4 \cdot C^4 (1 - p)^3$$

$$\frac{{}^7C_3}{5 \times {}^7C_4} = \frac{p}{1 - p}$$

$$1 - p = 5p$$

$$6p = 1$$

$$\frac{p}{1 - p} \cdot p = \frac{1}{6} \Rightarrow q = \frac{5}{6}$$

$$n = 7$$

$$\text{Mean} = np = 7 \times \frac{1}{6} = \frac{7}{6}$$

$$\text{Var} = npq = 7 \times \frac{1}{6} \times \frac{5}{6} = \frac{35}{36}$$

Sum

$$= \frac{7}{6} + \frac{35}{36}$$

$$= \frac{42 + 35}{36}$$

$$= \frac{77}{36}$$

18. The value of $\cos\left(\frac{2\pi}{7}\right) + \cos\left(\frac{4\pi}{7}\right) + \cos\left(\frac{6\pi}{7}\right)$ is equal to :

- (A) -1 (B) $-\frac{1}{2}$ (C) $-\frac{1}{3}$ (D) $-\frac{1}{4}$

Ans. (B)

Sol. $\cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{6\pi}{7}$

$$= \frac{\sin\left(3 \times \frac{\pi}{7}\right)}{\sin\frac{\pi}{7}} \times \cos\left(\frac{\frac{2\pi}{7} + \frac{6\pi}{7}}{2}\right)$$

$$= \frac{2\sin\left(\frac{3\pi}{7}\right)}{2\sin\frac{\pi}{7}} \times \cos\left(\frac{4\pi}{7}\right)$$

$$= \frac{2\sin\left(\frac{7\pi}{7}\right) + \sin\left(\frac{-\pi}{7}\right)}{2\sin\frac{\pi}{7}}$$

$$= \frac{-\sin\frac{\pi}{7}}{2\sin\frac{\pi}{7}}$$

$$= -\frac{1}{2}$$

19. $\sin^{-1}\left(\sin\frac{2\pi}{3}\right) + \cos^{-1}\left(\cos\frac{7\pi}{6}\right) + \tan^{-1}\left(\tan\frac{3\pi}{4}\right)$ is equal to :

- (A) $\frac{11\pi}{12}$ (B) $\frac{17\pi}{12}$ (C) $\frac{31\pi}{12}$ (D) $-\frac{3\pi}{4}$

Ans. (A)

Sol. $\sin^{-1}\left(\sin\frac{2\pi}{3}\right) + \cos^{-1}\left(\cos\frac{7\pi}{6}\right) + \tan^{-1}\tan\left(\frac{3\pi}{4}\right)$

$$\sin^{-1}\sin\left(\frac{2\pi}{3}\right) = \pi \frac{2\pi}{3} = \frac{\pi}{3}$$

$$\cos^{-1}\left(\cos\frac{7\pi}{6}\right) = 2\pi \frac{7\pi}{6} = \frac{5\pi}{6}$$

$$\tan^{-1}\tan\left(\frac{3\pi}{4}\right) = \frac{3\pi}{4} - \pi = \frac{-\pi}{4}$$

$$\sin^{-1}\left(\sin\frac{2\pi}{3}\right) + \cos^{-1}\cos\frac{7\pi}{6} + \tan^{-1}\tan\frac{3\pi}{4}$$

$$= \frac{11\pi}{12}$$

20. The Boolean expression $(\sim(p \wedge q)) \vee q$ is equivalent to :

- (A) $q \rightarrow (p \wedge q)$ (B) $p \rightarrow q$ (C) $p \rightarrow (p \rightarrow q)$ (D) $p \rightarrow (p \vee q)$

Ans. (D)

Sol. $(\sim(p \wedge q)) \vee q$
 $= (\sim p \vee \sim q) \vee q$
 $= \sim p \vee \sim q \vee q$
 $= \sim p \vee t$
 = this statement is a tautology option D
 $p \Rightarrow (p \vee q)$ is also a tautology.

OR

p	q	$p \wedge q$	$\sim(p \wedge q)$	$\sim(p \wedge q) \vee q$	$p \vee q$	$p \rightarrow (p \vee q)$
T	T	T	F	T	T	T
T	F	F	T	T	T	T
F	T	F	T	T	T	T
F	F	F	T	T	F	T

SECTION-B

1. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a function defined $f(x) = \frac{2e^{2x}}{e^{2x} + e}$. Then $f\left(\frac{1}{100}\right) + f\left(\frac{2}{100}\right) + f\left(\frac{3}{100}\right) + \dots + f\left(\frac{99}{100}\right)$ is equal to _____.

Ans. (99)

Sol. $f(x) + f(1-x) = \frac{2e^{2x}}{e^{2x} + e} + \frac{2e^{2-2x}}{e^{2-e^x} + e} \left[\frac{e^{2x}}{e^{2x} + e} + \frac{e^2}{e^2 + e^{2x+1}} \right]$

$$= 2 \left[\frac{e^{2x-1}}{e^{2x-1} + 1} + \frac{1}{1 + e^{2x-1}} \right] = 2$$

$$f\left(\frac{1}{100}\right) + f\left(\frac{2}{100}\right) + f\left(\frac{3}{100}\right) + \dots + f\left(\frac{99}{100}\right)$$

$$= \left\{ f\left(\frac{1}{100}\right) + f\left(\frac{99}{100}\right) \right\} + \left\{ f\left(\frac{2}{100}\right) + f\left(\frac{98}{100}\right) \right\} + \dots + f\left\{ \left(\frac{49}{100}\right) + f\left(\frac{51}{100}\right) \right\} + f\left(\frac{1}{2}\right)$$

$$= (2 + 2 + 2 + \dots + 49 \text{ times}) + \frac{2e}{e + e}$$

$$= 98 + 1 = 99$$

2. If the sum of all the roots of the equation $e^{2x} - 11e^x - 45e^{-x} + \frac{81}{2} = 0$ is $\log_e P$, is equal to _____.

Ans. (45)

Sol. $e^{2x} - 11e^x - 45e^{-x} + \frac{81}{2} = 0$

$$(e^x)^3 - 11(e^x)^2 - 45 + \frac{81e^x}{2} = 0$$

$$e^x = t$$

$$2t^3 - 22t^2 + 81t - 90 = 0$$

$$t_1 t_2 t_3 = 45$$

$$e^{x_1} \cdot e^{x_2} \cdot e^{x_3} = 45$$

$$e^{x_1 + x_2 + x_3} = 45$$

$$\log_e e^{x_1 + x_2 + x_3} = \log_e 45$$

$$x_1 + x_2 + x_3 = \log_e 45$$

$$\log_e P = \log_e 45$$

$$P = 45$$

3. The positive value of the determinant of the matrix A, whose $\text{Adj}(\text{Adj}(A)) = \begin{pmatrix} 14 & 28 & -14 \\ -14 & 14 & 28 \\ 28 & -14 & 14 \end{pmatrix}$ is

Ans. (14)

Sol. $\text{Adj}(\text{Adj}(A)) = \begin{bmatrix} 14 & 18 & -14 \\ -14 & 14 & 28 \\ 28 & -14 & 14 \end{bmatrix}$

$$|\text{Adj}(\text{Adj}A)| = \begin{vmatrix} 14 & 18 & -14 \\ -14 & 14 & 28 \\ 28 & -14 & 14 \end{vmatrix} = 14 \times 14 \times 14 \begin{vmatrix} 1 & 2 & -1 \\ -1 & 1 & 2 \\ 2 & -1 & 1 \end{vmatrix}$$

$$= (14)^3 [3 - 2(-5) - 1(-1)] = (14)^3 [14] = (14)^4$$

4. $|A|^4 = (14)^4 \Rightarrow |A| = 14$
 The number of ways, 16 identical cubes, of which 11 are blue and rest are red, can be placed in a row so that between any two red cubes there should be at least 2 blue cubes, is _____.

Ans. (56)

Sol. 16 cubes $\begin{cases} 11 \text{ Blue} \\ 5 \text{ Red} \end{cases}$

$$\begin{aligned} x_1 + x_2 + x_3 + x_4 + x_5 + x_6 &= 11 \\ x_1, x_6 &\geq 0, \quad x_2, x_3, x_4, x_5 \geq 2 \\ x_2 &= t_1 + 2 \\ x_3 &= t_2 + 2 \\ x_4 &= t_3 + 2 \\ x_5 &= t_4 + 2 \\ x_1, t_1, t_2, t_3, t_4, x_6 &\geq 0 \\ \text{No. of solutions} &= {}^{6+3-1}C_3 = {}^8C_3 = 56 \end{aligned}$$

5. If the coefficient of x^{10} in the binomial expansion of $\left(\frac{\sqrt{x}}{5^{\frac{1}{4}}} + \frac{\sqrt{5}}{x^{\frac{1}{3}}}\right)^{60}$ is 5^k , where $l, k \in \mathbb{N}$ and l is co-prime to 5, then k is equal to _____.

Ans. (5)

Sol. $\left(\frac{\sqrt{x}}{5^{\frac{1}{4}}} + \frac{\sqrt{5}}{x^{\frac{1}{3}}}\right)^{60}$

$$T_{r+1} = {}^{60}C_r \left(\frac{x^{\frac{1}{2}}}{5^{\frac{1}{4}}}\right)^{60-r} \left(\frac{5^{\frac{1}{2}}}{x^{\frac{1}{3}}}\right)^r$$

$$= {}^{60}C_r 5^{\frac{3r-60}{4}} \cdot x^{\frac{180-5r}{6}}$$

$$\frac{180-5r}{6} = 10 \Rightarrow r = 24$$

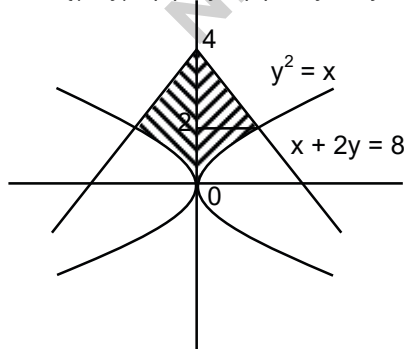
$$\text{Coeff. of } x^{10} = {}^{60}C_{24} \cdot 5^3 = \frac{60!}{24!36!} 5^3$$

$$\text{Powers of 5 in } {}^{60}C_{24} \cdot 5^3 = \frac{5^{14}}{5^4 \times 5^8} \times 5^3 = 5^5$$

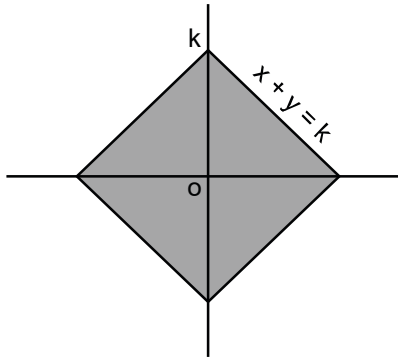
6. Let $A_1 = \{(x, y) : |x| \leq y^2, |x| + 2y \leq 8\}$ and $A_2 = \{(x, y) : |x| + |y| \leq k\}$. If $27 (\text{Area } A_1) = 5 (\text{Area } A_2)$, then k is equal to :

Ans. (6)

Sol. $A_1 = \{(x, y) : |x| \leq y^2, |x| + 2y \leq 8\}$ and $A_2 = \{(x, y) : |x| + |y| \leq k\}$.



$$\begin{aligned} \text{area}(A_1) &= 2 \left[\int_0^2 y^2 dy + \int_2^4 (8 - 2y) dy \right] \\ &= 2 \left[\left(\frac{y^3}{3} \right)_0^2 + (8y - y^2)_2^4 \right] \end{aligned}$$



$$\text{area}(A_1) = 2 \times \frac{20}{3} = \frac{40}{3}$$

$$\text{Area}(A_2) = 4 \times \frac{1}{2} k^2$$

$$\text{Area}(A_2) = 2k^2$$

Now

$$27 (\text{Area } A_1) = 5 (\text{Area } A_2)$$

$$9 \times 4 = k^2$$

$$k = 6$$

7. If the sum of the first ten terms of the series $\frac{1}{5} + \frac{2}{65} + \frac{3}{325} + \frac{4}{1025} + \frac{5}{2501} + \dots$ is $\frac{m}{n}$, where m and n are co-prime numbers, then m + n is equal to _____.

Ans. (276)

Sol. $\frac{1}{5} + \frac{2}{65} + \frac{3}{325} + \frac{4}{1025} + \frac{5}{2501} + \dots$

$$T_n = \frac{n}{4n^4 + 1}$$

$$= \frac{n}{(2n^2 + 1)^2 - (2n)^2} = \frac{n}{(2n^2 + 2n + 1)(2n^2 - 2n + 1)}$$

$$= \frac{1}{4} \left[\frac{1}{2n^2 - 2n + 1} - \frac{1}{2n^2 + 2n + 1} \right]$$

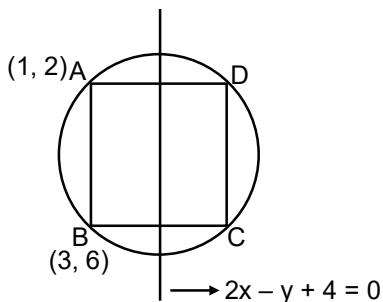
$$S_{10} = \sum_{n=1}^{10} T_n = \frac{1}{4} \left[\frac{1}{1} - \frac{1}{5} + \frac{1}{5} - \frac{1}{13} + \dots + \frac{1}{200 + 20 + 1} \right]$$

$$= \frac{1}{4} \left[1 - \frac{1}{221} \right] = \frac{1}{4} \times \frac{220}{221} = \frac{55}{221} = \frac{m}{n}$$

$$m + n = 55 + 221 = 276$$

8. A rectangle R with end points of the one of its sides as (1, 2) and (3, 6) is inscribed in a circle. If the equation of a diameter of the circle is $2x - y + 4 = 0$, then the area of R is _____.

Ans. (16)



Sol.

Eq. of line AB
 $y = 2x$
 Slope of given diameter = 2
 So that diameter is parallel to AB
 Distance between diameter and line AB

$$= \left(\frac{4}{\sqrt{2^2 + 12}} \right) = \frac{4}{\sqrt{5}}$$

Thus $BC = 2 \times \frac{4}{\sqrt{5}} = \frac{8}{\sqrt{5}}$

$$AB = \sqrt{(1-3)^2 + (2-6)^2} = \sqrt{20} = 2\sqrt{5}$$

$$\text{Area} = AB \times BC = \frac{8}{\sqrt{5}} \times 2\sqrt{5} = 16 \text{ Ans.}$$

9. A circle of radius 2 unit passes through the vertex and the focus of the parabola $y^2 = 2x$ and touches the parabola $y = \left(x - \frac{1}{4}\right)^2 + \alpha$, where $\alpha > 0$. Then $(4\alpha - 8)^2$ is equal to _____.

Ans. (63)

Sol. Vertex and focus of parabola $y^2 = 2x$ are V (0, 0) and S $\left(\frac{1}{2}, 0\right)$ resp.

Let equation of circle be
 $(x - h)^2 + (y - k)^2 = 4$

\therefore Circle passes through (0, 0)
 $\Rightarrow h^2 + k^2 = 4 \dots\dots (1)$

\therefore Circle passes through $\left(\frac{1}{2}, 0\right)$

$$\left(\frac{1}{2} - h\right)^2 + k^2 = 4$$

$$\Rightarrow h^2 + k^2 - h = \frac{15}{4} \dots\dots (2)$$

On solving (1) and (2)

$$4 - h = \frac{15}{4}$$

$$h = 4 - \frac{15}{4} = \frac{1}{4}$$

$$k = + \frac{\sqrt{63}}{4}$$

$k = - \frac{\sqrt{63}}{4}$ is rejected as circle with centre

$\left(\frac{1}{4}, -\frac{\sqrt{63}}{4}\right)$ can't touch given parabola.

Equation of circle is

$$\left(x - \frac{1}{4}\right)^2 + \left(k - \frac{\sqrt{63}}{4}\right)^2 = 4$$

From figure

$$\alpha = 2 + \frac{\sqrt{63}}{4} = \frac{8 + \sqrt{63}}{4}$$

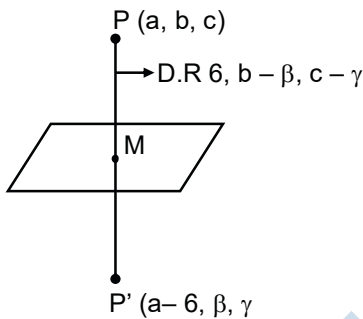
$$4\alpha - 8 = \sqrt{63}$$

$$(4\alpha - 8)^2 = 63$$

10 Let the mirror image of the point (a, b, c) with respect to the plane $3x - 4y + 12z + 19 = 0$ be $(a - 6, \beta, \gamma)$. If $a + b + c = 5$, then $7\beta - 9\gamma$ is equal to _____.

Ans. (137)

Sol.



$$M = \left(a - 3, \frac{\beta + b}{2}, \frac{\gamma + c}{2}\right)$$

Since M lies on $3x + 4y + 12z + 19 = 0$

$$\Rightarrow 6a - 4b + 12c - 4\beta + 12\gamma + 20 = 0 \dots\dots (1)$$

Since PP' is parallel to normal of the plane then

$$\frac{6}{3} = \frac{b - \beta}{-4} = \frac{c - \gamma}{12}$$

$$\Rightarrow \beta = b + 8, \gamma = c - 24$$

$$a + b + c = 5 \Rightarrow a + \beta - 8 + \gamma + 24 = 5$$

$$\Rightarrow a = -\beta - \gamma - 11$$

Now putting these values in (1) we get

$$6(-\beta - \gamma - 11) - 4(\beta - 8) + 12(\gamma + 24) - 4\beta + 12\gamma + 20 = 0$$

$$\Rightarrow 7\beta - 9\gamma = 170 - 33 = 137$$